

Problem Statement:

$$\max x_1(2\pi) = \min -x_1(2\pi)$$

subject to dynamics:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} - \begin{pmatrix} x_2 \\ \cos t - x_2 - kx_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

periodic boundary conditions:

$$\begin{pmatrix} x_1(2\pi) - x_1(0) \\ x_2(2\pi) - x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

and phase condition

$$x_2(0) = 0 \text{ to ensure that } x_1(0) = x_1(2\pi) \text{ is a maximum}$$

Allow the linear stiffness term, k , to vary. Using adjoint formulas in ENOC
Submission:

$$\delta\lambda_1: \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} - \begin{pmatrix} x_2 \\ \cos t - x_2 - kx_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\delta\eta_2: k - \mu_2 = 0$$

$$\delta\lambda_2: \begin{pmatrix} x_1(2\pi) - x_1(0) \\ x_2(2\pi) - x_2(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\delta\eta_1: -x(2\pi) - \mu_1 = 0$$

Adjoint Equations:

$$\delta x: -(\dot{\lambda}_{1,1} \quad \dot{\lambda}_{1,2}) - (\lambda_{1,1} \quad \lambda_{1,2}) \begin{pmatrix} 0 & 1 \\ -k & -1 \end{pmatrix} = (0 \quad 0);$$

$$\delta x(0): (\lambda_{1,1}(0), \lambda_{1,2}(0)) + \lambda_2(-1 \quad 0) = 0; (x_2(0) \text{ doesn't vary})$$

$$\delta x(1): (\lambda_{1,1}(1), \lambda_{1,2}(1)) + \lambda_2(1 \quad 0) + (1 \quad 0) = 0; (x_2(1) \text{ doesn't vary})$$

$$\delta p: -(\lambda_{1,1}, \lambda_{1,2}) \begin{pmatrix} 0 \\ -x_1 \end{pmatrix} = 0$$

Solution:

Ignoring transients, and focusing on periodic solutions of the differential equations yields

$$x_1(t) = \frac{1}{\sqrt{(k-1)^2+1}} \cos(t)$$